$$z = f(x,y) = -x^2 - y^2 + 4,$$

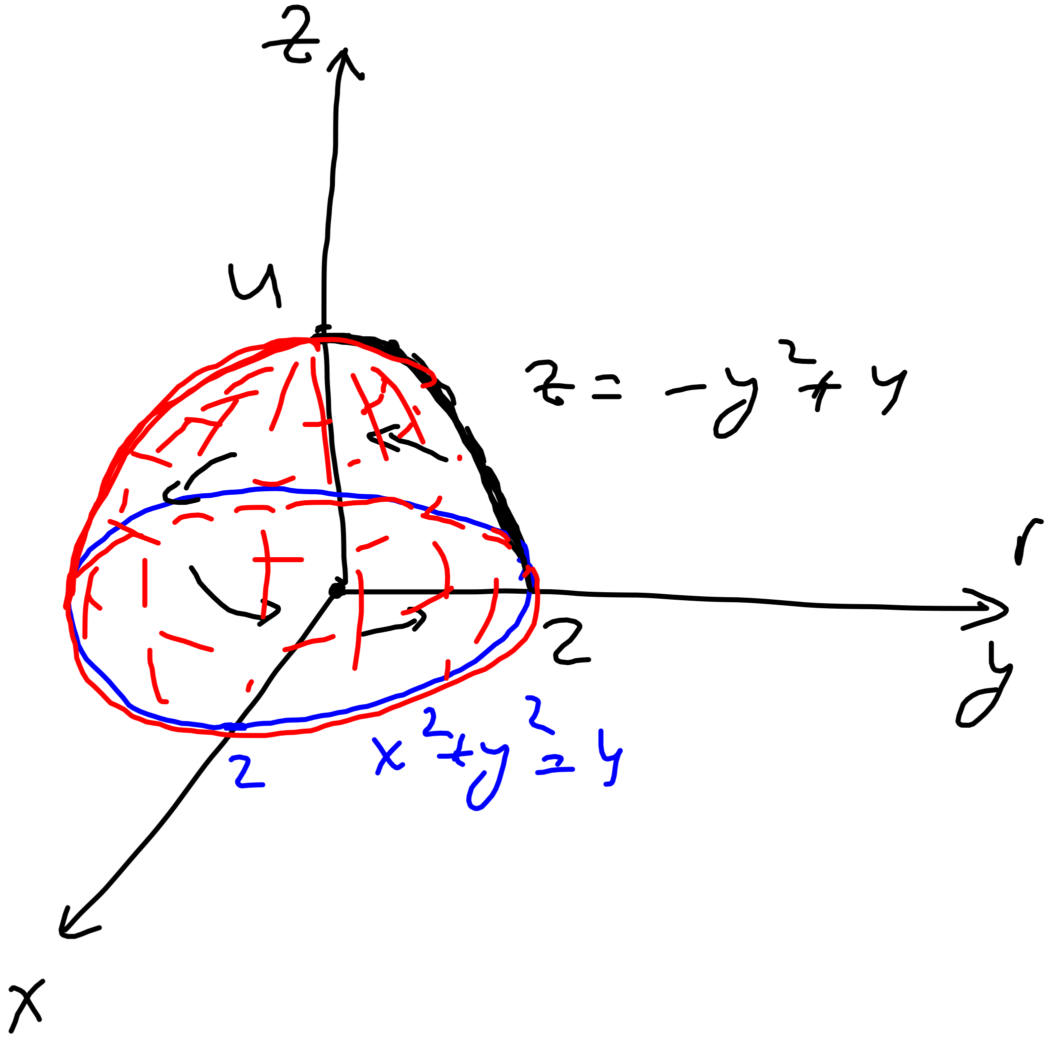
$$x^2 + y^2 \leq 4$$

$$(x^2 + y^2) =: r^2;$$

$$f(x,y) = f(r) = -r^2 + 4 =$$

$$z$$

$$z = -r^2 + 4$$



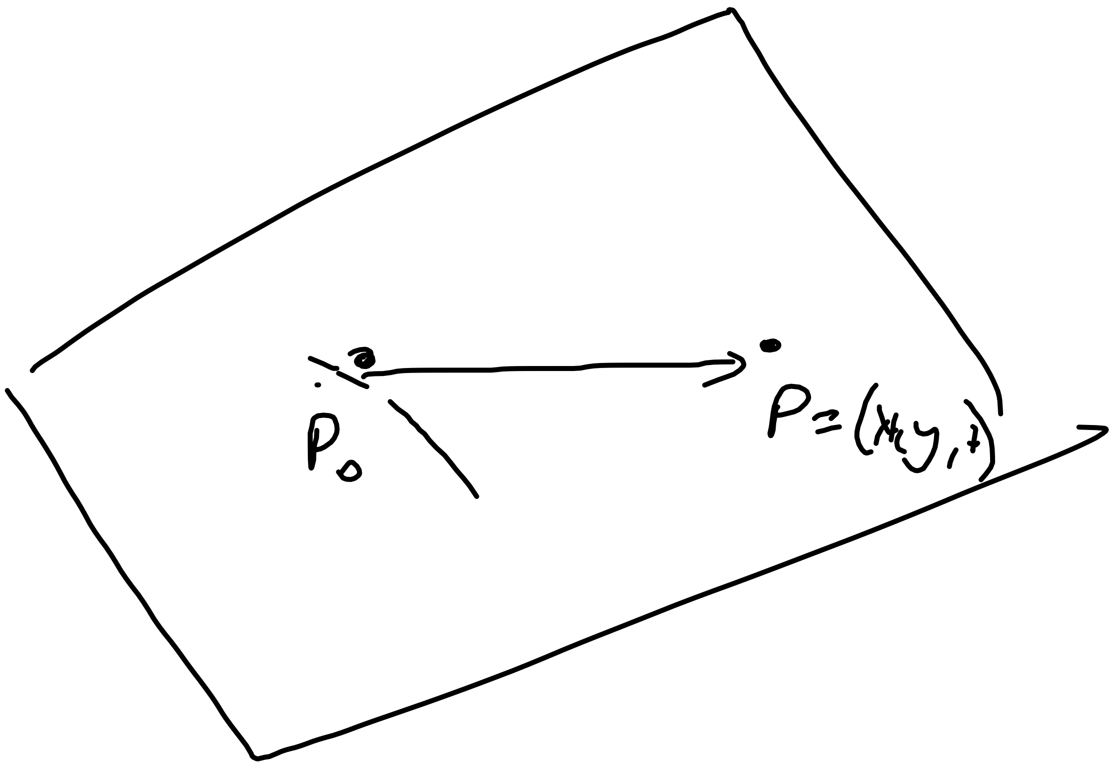
$$x^2 + y^2 \leq y$$

$$y \geq x^2 + y^2 \geq y^2 \Longrightarrow \begin{array}{l} x \leq 2 \\ y \leq 2 \end{array}$$

$$P_0 = (1,1)$$

$$t_{P_0}(x,y) = z$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{array}{l} \leftarrow \text{Unab. Var} \\ \leftarrow \\ \leftarrow \text{Abh. Var} \end{array}$$



$f(x)$

(a, b) $\qquad x_0 = a$

$f(x_0) = b$

$t: f(x_0) + f'(x)(x - x_0)$

$$z = t_p(x, y) = f(P_0) +$$

$$+ \left(\overrightarrow{\text{grad}}(P_0) \,;\, \overrightarrow{P_0 P} \right) =$$

Skalarprodukt

$$\boxed{= f(P_0) + \overrightarrow{\text{grad}}(P_0)^T \cdot \overrightarrow{P_0 P}}$$

$$\bar{a} \cdot \bar{b} = \langle \bar{a}, \bar{b} \rangle =$$

$$= (\bar{a}, \bar{b}) =$$

$$= \sum_{i=1}^{n} a_i b_i ,$$

$$a, b \in \mathbb{R}^n$$

Skalarprodukt

"Zeilenvektor · Spaltenvektor"

$\Leftarrow$ als Matrizenprodukt

$$-x^2-y^2+4 = f(x,y)$$

$$\overrightarrow{grad}(f) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$

$$\frac{\partial f}{\partial x} = \frac{\partial(-x^2-y^2+4)}{\partial x} =$$

$$= -2x;$$

$$\frac{\partial f}{\partial y} = -2y;$$

$$\overrightarrow{\operatorname{grad}}\, f = \begin{pmatrix} -2x \\ -2y \end{pmatrix}$$

$$P_0 = (1,1)$$

$$\overline{\operatorname{grad}}\, f(P_0) = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$z = t_{P_0}(x,y) = \overbrace{\left(-1^2 - 1^2 + 4\right)}^{f(P_0)} +$$

$$+ \begin{pmatrix} -2 & -2 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-1 \end{pmatrix} =$$

$$= 2 + \left(-2(x-1) + (-2)(y-1)\right)$$

$$= 6 - 2x - 2y$$

Tang.-Ebene:

$$\boxed{z = 6 - 2x - 2y}$$

b) $P_0 \ (1,1)$

$$\bar{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Richtungsvektor $\bar{b}$

Aufpunkt Q_0 :

$$g = \{ Q \mid \underset{\bar{r}(Q)}{Q} = \underset{\bar{r}(Q_0)}{Q_0} + \lambda \bar{b} \}$$

$$\boxed{P = P_0 + \lambda \bar{a}} \in t_{P_0}(x,y)$$

$$t_{\bar{a},P_0}(x,y) = f(P_0) + \overrightarrow{\text{grad}} f(P_0)^T \cdot \overline{P_0 P}$$

$$= f(P_0) + \overrightarrow{\text{grad}} f(P_0)^T \cdot \lambda \bar{a}$$

$$= 2 + \lambda(-2; -2)\begin{pmatrix}1\\1\end{pmatrix} =$$

$$= 2 - 4\lambda = z$$

$$x: \quad x(P_0) + \lambda a_1 = 1 + \lambda \cdot 1$$

$$y: \quad y(P_0) + \lambda a_2 = 1 + \lambda \cdot 1$$

$$z = \quad 2 - 4\lambda$$

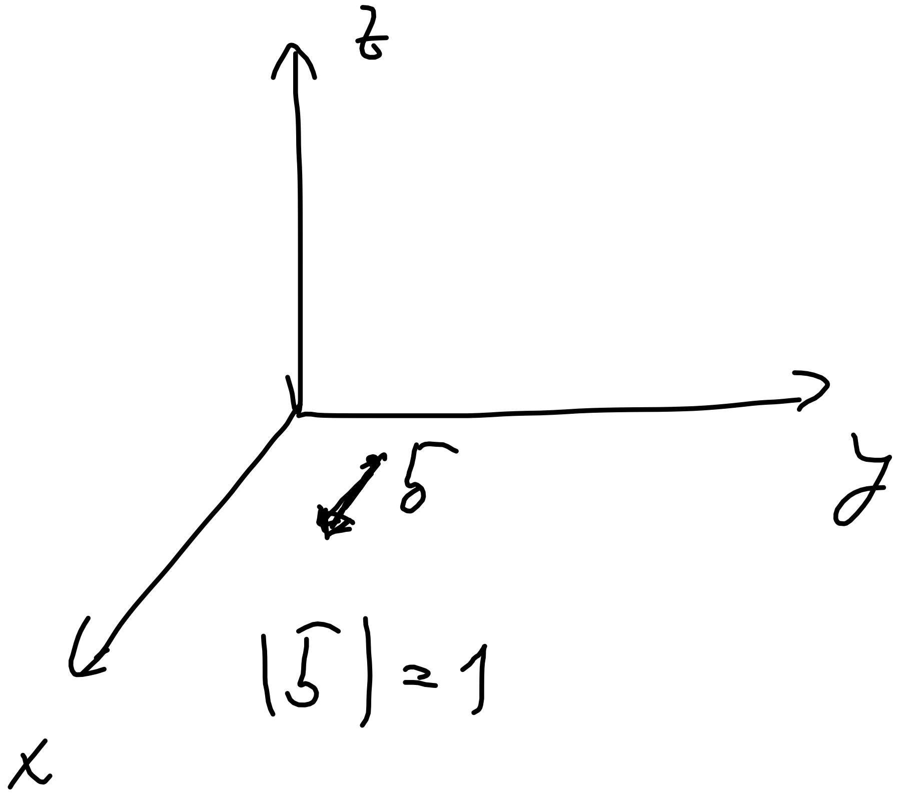
$$F(\lambda) = \begin{pmatrix}1+\lambda\\1+\lambda\\2-4\lambda\end{pmatrix} = \begin{pmatrix}1\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\-4\end{pmatrix}$$

d)

$$d f(P_0) = \frac{\partial f}{\partial x}(P_0) \cdot dx + \frac{\partial f}{\partial y}(P_0)\, dy =$$

$$= \left\langle \begin{pmatrix} \frac{\partial f}{\partial x}(P_0) \\ \frac{\partial f}{\partial y}(P_0) \end{pmatrix}, \begin{pmatrix} dx \\ dy \end{pmatrix} \right\rangle$$

$$= \overrightarrow{\text{grad}} f(P_0)^T \cdot \begin{pmatrix} dx \\ dy \end{pmatrix}$$



$|\vec{s}| = 1$

x_0

y_0

$$\Rightarrow \quad df(P_0) = (-2;\ -2)\begin{pmatrix} dx \\ dy \end{pmatrix} = -2\,dx - 2\,dy$$